PHOTOFISSION AND QUASI-DEUTERON-NUCLEAR STATE AS MIXING OF BOSONS AND FERMIONS *

G. Kaniadakis, A. Lavagno † and P. Quarati

Dipartimento di Fisica - INFM - Politecnico di Torino - 10129 Torino, Italy

Istituto Nazionale di Fisica Nucleare - Sezione di Cagliari and Sezione di Torino

The empirical-phenomenological quasi-deuteron photofission description is theoretically justified within the semiclassical, intermediate statistics model. The transmutational fermion (nucleon) - boson (quasi-deuteron) potential plays an essential role in the present context and is expressed in terms of thermodynamical and of microscopical quantities, analogous to those commonly used in the superfluid nuclear model.

PACS number(s): 25.85.Jg, 02.50.Ga

I. INTRODUCTION

Over the past several years, many efforts have been directed to develop methods for the physical description of complex many-body systems. The implementation of quantum approaches like the Hartree-Fock, the Random Phase Approximation and the Time Dependent Hatree-Fock Method have allowed the explanation of a great variety of experimental results and particularly of the properties of the nuclear ground state [1,2].

In the literature, the nuclear dynamics is described by different theories. In the most cases, approaches based on semiclassical one-body equations (Fokker-Plank, Boltzmann, Boltzmann-Langevin equations) are use. These methods can be regarded as semiclassical analogs of the mentioned quantum approaches [3,4] and include the Pauli exclusion and the inclusion principle [5] to represent correctly systems of fermions, bosons or anyons.

In several problems in condensed matter and in nuclear physics, the correlation effects between pairs of fermions are quite relevant in the interpretation of experimental results. Similarly, the interactions among bosons are relevant to the superfluid nuclear model, the interacting boson model and the mean field boson approximation and allow the explanation of many collective nuclear properties. In these cases, the interaction among the valence nucleons outside the core produces pairs of correlated nucleons that can be approximated with bosonic particles (like, for instance, the quasi-deuterons). The pairing of fermions leads a certain amount of nucleons to a transition from one statistics to another (e.g. Bose-Einstein). This transition can be modeled with a transmutational potential (TP) and implies that a nucleon can be considered, in particular excited states, as an ensemble of fermions and bosons or as an ensemble of particles of intermediate statistics.

Semiclassical kinetic approaches to intermediate statistics [5-7] are based on the classical Fokker-Planck equation and describe the time evolution of the distribution function $n(t, \mathbf{v})$. The Fokker-Planck equation is corrected with the insertion of the factor $1 + \kappa n(t, \mathbf{v})$ into the transition rates, where κ varies between -1 (Fermi-Dirac distribution (FD)) and $+1$ (Bose-Einstein distribution (BE)), ($\kappa = 0$ is the case of the Maxwell-Boltzmann distribution). The effect on the distribution function of the exclusion ($\kappa < 0$) and of the inclusion ($\kappa > 0$) principle, can be evaluated with this approach.

This approach is not unique, different non linear expressions can also be considered [7]. However, more involved expressions are not required for the discussion presented in this work.

In the present paper, we call statistical transmutation the variation of the particle state from its characteristic value κ to a new value κ' (this definition includes, in particular, the transmutation from $\kappa = -1$ to $\kappa' = +1$).

To describe the importance of the TP, we fix our attention to the photofission of heavy nuclei in the energy range of excitation between 40 MeV and 100 MeV (usually called quasi-deuteron (QD) energy region [8,9]).

In photonuclear reactions, it is well known that, above the GDR and below the pion photoproduction threshold, the photoabsorption involves two nucleons to satisfy the kinematical conservation laws. This circumstance helps the formation of strongly correlated pairs of nucleons (quasi-deuteron states). The photoabsorption cross section is proportional to the deuteron photodisintegration cross section.

---

*To appear in Zeit. f. Phys. A

†e-mail: alavagno@polito.it

Recently, we have developed an extension of the QD model, to describe the total photoabsortion and the photofission of heavy nuclei [10-11] (an extension to higher excitation energies, pion photoproduction region, of the QD photofission model has been recently developed by Arruda-Neto and collaborators [12,13]). The agreement of the model with the experimental results is quite good for a wide variety of nuclei (Bi, Th, U) (see also Ref.s [14-16]). Our phenomenological-empirical model provides us useful quantities, such as the number of QD participating to photofission at different excitation energies. Its use, together with the semiclassical kinetic approach, provides a theoretical basis for the phenomenological model itself. The evaluation of the TP in analogy with the superfluid model, allows the introduction of an analytical expression for the excited QD nuclear state as a mixture of bosons and fermions. The TP can be given an interpretation in terms of the occupation operators. An interpretation of the TP in terms of thermodynamical quantities is also given.

In Sect. II, we describe the equations of the transmutational kinetic model and provide the free nucleon fraction and the probability of participation of one nucleon to a QD pair.
In Sect. III, we calculate the transmutational potential using the results of a phenomenological approach to photofission, valid in the QD energy region and expressed in terms of the nuclear latent heat.
In Sect. IV, we give an interpretation of the transmutational potential using an analogy with the superfluid model of the nucleus. Conclusion are outlined in Sect. V.

II. NUCLEON-QUASI DEUTERON TRANSMUTATION MODEL

The protons and the neutrons are distinguished by their third isospin component $t_3 = \tau$ ($\bar{\tau} = -\tau$). The nucleon mass is indicated by $m_{_N} = 939\ MeV$ and the occupational number in the velocity space by $n^{\tau}_{_N}(t, \mathbf{v})$. The QD particles, neglecting their binding energy, have mass $m_{_D} = 2\, m_{_N}$, $L = 0$, $S = 1$, $T = 0$. Their occupational number is $n_{_D}(t, \mathbf{v})$.
By using an intermediate statistics with a linear enhancement or inhibition factor to calculate the QD photofission cross section, we can assign to the nucleons a value of the parameter κ, intermediate between -1 and 1 and different from zero. When the excitation energy increases, the parameter κ varies because the QD particles are more numerous as the nucleus is more excited. Therefore, the bosonic component of each nucleon increases and one can study the TP from the statistical transmutation from κ to κ'. We are interested in the transmutation nucleon-QD or viceversa, therefore we must fix, in this context, the states κ and κ' as the fermion and boson state, respectively.
An alternative, but equivalent, procedure is to assume that the system (nucleus) is composed by a certain number of fermions (nucleons) and by a certain number of bosons (QD), depending on the excitation energy. Therefore, we write two different kinetic equations for the nucleons, with $\kappa = -1$, and for the QD, with $\kappa = +1$. In this way it can be clearly understood the role of the fermion-boson TP.

We consider the energy as a continuous variable, the two functions $n^{\tau}_{_N}(t, \mathbf{v})$ and $n_{_D}(t, \mathbf{v})$ obey the following system of coupled equations [6]:

$$\frac{\partial n^{\tau}_{_N}(t, \mathbf{v})}{\partial t} + \nabla \mathbf{j}^{\tau}_{_N}(t, \mathbf{v}) + j^{\tau\bar{\tau}}_{_{2N \leftrightarrow D}}(t, \mathbf{v}) = 0 \quad , \tag{1}$$

$$\frac{\partial n_{_D}(t, \mathbf{v})}{\partial t} + \nabla \mathbf{j}_{_D}(t, \mathbf{v}) + j^{\tau\bar{\tau}}_{_{D \leftrightarrow 2N}}(t, \mathbf{v}) = 0 \quad . \tag{2}$$

These equations can be viewed as the continuity equations of a nucleon and of a quasi-deuteron gas in the velocity space. When the energy is a discrete variable, we can substitute Eq.s (1) and (2) with a system of coupled master equations.
In the velocity space, the currents $\mathbf{j}^{\tau}_{_N}(t, \mathbf{v})$ and $\mathbf{j}_{_D}(t, \mathbf{v})$ have the following expressions:

$$\mathbf{j}^{\tau}_{_N}(t, \mathbf{v}) = -\left[\mathbf{J}^{\tau}_{_N}(t, \mathbf{v}) + \nabla D^{\tau}_{_N}(t, \mathbf{v})\right] n^{\tau}_{_N}(t, \mathbf{v})[1 - n^{\tau}_{_N}(t, \mathbf{v})] - D^{\tau}_{_N}(t, \mathbf{v}) \nabla n^{\tau}_{_N}(t, \mathbf{v}) \quad , \tag{3}$$

$$\mathbf{j}_{_D}(t, \mathbf{v}) = -\left[\mathbf{J}_{_D}(t, \mathbf{v}) + \nabla D_{_D}(t, \mathbf{v})\right] n_{_D}(t, \mathbf{v})[1 + n_{_D}(t, \mathbf{v})] - D_{_N}(t, \mathbf{v}) \nabla n_{_D}(t, \mathbf{v}) \quad , \tag{4}$$

where $\mathbf{J}_{_{N,D}}(t, \mathbf{v})$ and $D_{_{N,D}}(t, \mathbf{v})$ are, respectively, the drift and diffusion coefficients for the nucleon and the QD.
The net transmutational current $j^{\tau\bar{\tau}}_{_{2N \leftrightarrow D}}$ ($j^{\tau\bar{\tau}}_{_{D \leftrightarrow 2N}}$) takes into account the formation of a quasi-deuteron (two nucleons) from two nucleons (a deuteron) with different third isospin component; its expression, according to the exclusion-inclusion principle (EIP), is the following [6]:

$$j^{\tau\bar{\tau}}_{2N\leftrightarrow D}(t,\mathbf{v}) = r_{2N\to D}(t)\, g_N(t,\mathbf{v})\, n^{\tau}_N(t,\mathbf{v})\, n^{\bar{\tau}}_N(t,\mathbf{v})\,[1+n_D(t,\mathbf{v})]$$
$$-r_{D\to 2N}(t)\, g_D(t,\mathbf{v})\, n_D(t,\mathbf{v})\,[1-n^{\tau}_N(t,\mathbf{v})][1-n^{\bar{\tau}}_N(t,\mathbf{v})] \quad , \tag{5}$$

where $r_{2N\to D}(t)$ is the transmutation rate of two nucleons in a quasi-deuteron, $r_{D\to 2N}(t)$ is the transmutation rate of a quasi-deuteron into two nucleons, $g_N(t,\mathbf{v})$ and $g_D(t,\mathbf{v})$ are functions taking into account the interaction among the identical particles belonging to the bound system we are considering.
From the definition of the net transmutational current we have:

$$j^{\tau\bar{\tau}}_{2N\leftrightarrow D}(t,\mathbf{v}) = -j^{\tau\bar{\tau}}_{D\leftrightarrow 2N}(t,\mathbf{v}) \tag{6}$$

Equations (1-6) define univocally the diffusion process of neutrons, protons and quasi-deuterons in the velocity space and the formation and disgregation of quasi-deuterons inside the nucleus.

The statistical distribution $n_{N,D}(\mathbf{v})$ can be obtained, in stationary conditions ($t\to\infty$), when the currents $\mathbf{j}^{\tau}_N(t,\mathbf{v})$ and $\mathbf{j}_D(t,\mathbf{v})$ vanish.
If $D_{N,D}(\mathbf{v}) = D_{N,D}(v)$ and $\mathbf{J}_{N,D}(\mathbf{v}) = \mathbf{v}\, J_{N,D}(v)/v$ we obtain:

$$n_{N,D}(\mathbf{v}) = \frac{1}{\exp[\beta(E_{N,D}-\mu_{N,D})] - \kappa_{N,D}} \quad , \tag{7}$$

where $E_{N,D} = \frac{1}{2} m_{N,D} v^2 + V_{N,D}(v)$ and

$$\frac{\partial V_{N,D}(v)}{\partial v} = \frac{1}{\beta D_{N,D}(v)}\left[J_{N,D}(v) + \frac{\partial D_{N,D}(v)}{\partial v}\right] - m_{N,D}v \quad ; \tag{8}$$

$\kappa_N = -1$, $\kappa_D = 1$, $\beta = 1/kT$ and T is the temperature of the system, $\mu_{N,D}$ is the chemical potential of the nucleons and the QD particles. In Eq. (8), $\partial V_{N,D}(v)/\partial v$ represents the difference between the force acting among the particles and the Brownian force $m_{N,D}v$.
We note that, in the case of Brownian particles, the potential $V_{N,D}(v)$ is a constant and $n_{N,D}(\mathbf{v})$ reproduces the standard FD and BE statistical distribution.
We define the transmutational potential $\eta_{2N\to D}$ as

$$\frac{r_{2N\to D}}{r_{D\to 2N}} = \exp[\beta\eta_{2N\to D}] \tag{9}$$

and introduce the potential $h(v)$

$$\frac{g_N(v)}{g_D(v)} = \exp[\beta h(v)] \quad . \tag{10}$$

The condition that the net transmutational current, given by Eq.(5), must satisfy the equation $j_{2N\leftrightarrow D}(\infty,\mathbf{v}) = 0$, imposes the two following relations:

$$\eta_{2N\to D} = \mu_D - 2\mu_N \quad , \tag{11}$$

$$h(v) = V^{\tau}_N(v) + V^{\bar{\tau}}_N(v) - V_D(v) \quad . \tag{12}$$

Equation (11), at any excitation energy E_γ, provides a relationship between the chemical potentials of the two different types of particles and the TP. Eq. (12) allows us to obtain the potential $h(v)$ in terms of the interaction potentials (constant in the case of brownian particles).
Let us define the two quantities:

$$\xi_{N,D} = \frac{N_{N,D}}{A} \quad , \tag{13}$$

which are, respectively, the free nucleon fraction and the quasi-deuteron fraction, A is the mass number of the nucleus. In other words: ξ_N and $2\xi_D$ represent the probability that a nucleon is a free nucleon or a part of a QD. ξ_N and ξ_D satisfy the normalization condition:

$$\xi_N + 2\xi_D = 1 \quad . \tag{14}$$

The fraction $\xi_{N,D}$ of nucleons and of QD is given by:

$$\xi_{N,D} = f_{N,D} \, \frac{1}{\rho} \left(\frac{m_{N,D}}{h}\right)^3 \int n_{N,D}(\mathbf{v}) d^3 v \quad , \tag{15}$$

where ρ is the nuclear density and $f_{N,D}$ is the spin-isospin degeneration factor ($f_N = 4$ for the nucleons; $f_D = 3$ for the QD, having total spin $S = 1$ and total isospin $T = 0$), V is the nuclear volume, h the Planck constant.
Note that we have considered pairs with $L = 0$, $S = 1$ and $T = 0$, that are the quantum numbers of a free deuteron. Recently, it has been considered the presence of QD pairs with $T = 1$ to obtain agreement with the experimental results on photoabsorption [17-19]. In our case, if we considered a QD component with $T = 1$ and $S = 1$, we should have taken the degeneration factor $f_D = 6$. In that case, the value of the QD chemical potential would be reduced, because the total number of particles is fixed by the Eq.(15) and also the value of the transmutational potential $\eta_{2N \to D}$ would be reduced because of Eq.(11). The insertion of this component is not relevant to the results obtained in this work.

If, for a given nucleus, ξ_D and ξ_N are known, as a function of the excitation energy, inverting Eq.(15), one can deduce the chemical potentials μ_N e μ_D and, by means of equation (11), derive the transmutational potential $\eta_{2N \to D}$ as a function of E_γ.

III. CALCULATION OF THE TRANSMUTATIONAL POTENTIAL

If the energy of the incident photon is fully converted into excitation energy, the nuclear temperature T can be calculated from the well known relation:

$$E_\gamma = \frac{A}{8} T^2 \text{ MeV} \quad . \tag{16}$$

In our case this relation is not rigorously exact, as shown by Montecarlo calculations in an intranuclear cascade model [20,21]. Indeed, not all the photon energy contributes to increase the nuclear temperature. In the QD energy region, the excitation energy is about 15% smaller than the photon energy [21]. We have verified that our results are not modified if we consider this reduction (for a discussion on the validity of Eq.(16) see Ref.[22]).

The QD and nucleon fractions have been determined using our QD model of photofission [10]. Let us recall that the following quantities. First,

$$C(N, Z) = 7.72 \, \frac{NZ}{A} \quad , \tag{17}$$

is the effective number of QD pairs used to write the photabsorption cross section as the product of the QD effective number times the photodisintegration cross section of the deuteron σ_D. Second,

$$F_1(E_\gamma) = \exp\left(-\frac{D}{E_\gamma}\right) \quad , \tag{18}$$

is the probability that one of the $C(N, Z)$ pairs takes part in the photoabsorption reaction (this factor is due to the Pauli blocking and the quantity $D/2$ is the average energy to excite each nucleon of a pair above the Fermi level ($D = 60 \, MeV$)). Third,

$$F_2(E_\gamma) = \exp\left(-\frac{D - \Gamma(E_\gamma)}{E_\gamma}\right) \left[1 - \exp\left(-\frac{D + \Gamma(E_\gamma)}{E_\gamma}\right)\right] \quad , \tag{19}$$

is the probability that one of the $C(N, Z)$ pairs participates in a photoabsorption evolving toward photofission (the product $C(N, Z) F_2 \sigma_D$ is the photofission cross section); $\Gamma(E_\gamma)$ is a polynomial in E_γ (we have introduced it to reduce, in the first factor of F_2, the photofission probability at low excitation energies, where the photoabsorption without fission is the most probable process, and to take into account, by means of the second factor, the photofission reduction at high energies). In the QD energy region, the contribution of the photabsorption to the continuum is greater than other contributions. $\Gamma(E_\gamma)$ represents the window of energy states, around the Fermi level, allowed to be occupied by the excited nucleons and pairs to arrange a system of particles evolving toward fission.
From Eq.s (13) and (14) the number of QD particles, correctly normalized is:

$$\xi_D(E_\gamma) = \frac{1}{2} F_2(E_\gamma) \quad , \tag{20}$$
$$\xi_N(E_\gamma) = 1 - F_2(E_\gamma) \quad . \tag{21}$$

We have considered three nuclei with very different photofission features: ^{209}Bi, ^{232}Th, ^{238}U.
In the Table the quantity $F_2(E_\gamma)$ for the three different nuclei is reported at different excitation energies. The numerical values are obtained from the functions $\Gamma(E_\gamma)$ given in Ref.[10].

We can calculate the QD and the nucleon fractions, at any excitation energy, with Eq.s (20,21). Using Eq.(15), we deduce the QD and nucleon chemical potentials and finally, using relation (11), the TP.
The nuclear density has been assumed equal to the infinite nuclear matter, i.e. $0.17\, nucleon\, fm^{-3}$.
The nuclear interaction of Eq.(12) is described by a square well with $V_N = V_D = V_0 = -48\ MeV$. At the low excitation temperature $T = 1.1 \div 1.9\ MeV$, the chemical potentials $\mu_D \approx V_0$, for ^{238}U and ^{232}Th and varies between $-55\ MeV$ and $-51\ MeV$ for ^{209}Bi.

The quantity $\eta_{2N\to D}(E_\gamma)$ is an increasing function of E_γ, as shown in the Figure. As the quantity of $\eta_{2N\to D}$ approaches a positive value, the number of quasi deuterons participating to the photofission becomes greater than the number of single nucleons. Infact, for ^{238}U, $\eta_{2N\to D}(90\, MeV) = 0$ and at this energy $F_2(90\, MeV) \approx 0.5$, which equals the value reported in the Table. Only in the case of ^{209}Bi, $\eta_{2N\to D}(E_\gamma)$ is nearly constant and negative. Let us note that, when $\eta_{2N\to D} \to -\infty$, the full system is composed by fermions (see Eq. (9)), on the other hand, if $\eta_{2N\to D} \to +\infty$ the full system is composed by bosons.

In ref. [10,11] we have shown that the photofission probability $P_f = P_f(E_\gamma)$ is the solution of the following equation

$$\frac{dP_f}{P_f} = f(\sqrt{E_\gamma})\, \frac{d\sqrt{E_\gamma}}{E_\gamma} \quad , \tag{22}$$

where $f(\sqrt{E_\gamma})$ is a polynomial in $\sqrt{E_\gamma}$.
When $f(\sqrt{E_\gamma})$ is nearly constant (as in the case of ^{209}Bi), the solution of Eq.(22) is given by the simple expression $P_f = a + b/\sqrt{E_\gamma}$ which is the result of the statistical model. Usually $f(\sqrt{E_\gamma})$ is not constant and the expression of P_f is more involved.
Eq.(22) can be seen as a generalized Clausius-Clapeyron equation where $f(\sqrt{E_\gamma})$ plays, in the photofission, the same role of the entalpy in the phase transition process.
We can assume that the function $f(\sqrt{E_\gamma})$ governs the first order phase transition from fermions (nucleons) to bosons (QD), capable of producing a compound system, which then proceeds to a saddle point, to scission and finally to fission.

We can define the nuclear latent heat dL of the transition process as

$$dL = \sqrt{E_\gamma}\, d\left(\frac{f(\sqrt{E_\gamma})}{\sqrt{E_\gamma}}\right) \quad . \tag{23}$$

From thermodynamic arguments and taking into account the transmutational potential defined in Eq.(9) and (11), the latent heat, at the thermodynamic equilibrium, can be written as

$$L = T\left[2\,\frac{S_N}{N_N} - \frac{S_D}{N_D} - \frac{d\eta_{2N\to D}}{dT}\right] \quad , \tag{24}$$

where S_N and S_D are, respectively, the entropies of the nucleons and the QD.
From Eq.(24), we can realize that the important physical quantity is the variation of the transmutational potential with the nuclear temperature T (or with $\sqrt{E_\gamma}$); infact, this quantity modifies the balance of the entropy of the two phases of the system. $\eta_{2N\to D}(T)$ is an increasing function (except for ^{209}Bi, where $\eta_{2N\to D}$ is nearly constant and the phase-transition is not relevant), therefore $-d\eta_{2N\to D}/dT$ decreases the latent heat, favoring the phase transition or, equivalently, increasing the photofission probability.

IV. MICROSCOPIC INTERPRETATION OF THE TRANSMUTATIONAL POTENTIAL

Let us now outline a microscopic interpretation of the transmutational potential. In the BCS theory, the ground state is composed by particles all paired with opposite spins:

$$|BCS>=\prod_{k>0}(u_k+v_k\hat{a}^\dagger_k\hat{a}^\dagger_{-k})|0> \quad , \tag{25}$$

with the normalization condition:

$$u_k^2+v_k^2=1 \quad . \tag{26}$$

In Eq.(25), v_k^2 and u_k^2 represent, respectively, the probability that the state k is occupied by a pair of particles or not. We recall that the antisymmetrization is contained in the anticommutation properties of the fermionic operators and the product operator applies only over the $k>0$ states. The states $k<0$ refers to the conjugate states having the third component of the spin with opposite values.

In our case, where QD pairs are contained into the nucleus, the state k can be empty or occupied by one non-paired nucleon or by a proton-neutron pair in a triple spin state. In analogy with Eq.(25), we can write the nuclear QD state at a given excitation energy in this form:

$$|NQD>=\prod_{k>0}(u_k+c_k\hat{a}^\dagger_k+\overset{S=1}{v_k}\hat{a}^\dagger_k\hat{a}^\dagger_{-k})|0> \tag{27}$$

where, now, $-k$ indicates the state with third isospin component having the opposite value.
The condition in Eq.(26) becomes:

$$u_k^2+c_k^2+v_k^2=1 \tag{28}$$

where u_k^2 is the probability that the state k is non-occupied; c_k^2 is the probability that the state k is occupied by one nucleon; v_k^2 is the probability that the state k is occupied by one QD pair.
The three coefficients u_k, c_k, v_k are functions of the excitation energy. With the above definitions we can write:

$$\frac{r_{2N\rightarrow D}}{r_{D\rightarrow 2N}}=\sum_k\left(\frac{v_k}{c_k}\right)^2 \tag{29}$$

and using the definition of the transmutational potential given in Eq. (9):

$$\eta_{2N\rightarrow D}(E_\gamma)=k_BT\log\left[\sum_k\left(\frac{v_k}{c_k}\right)^2\right] \quad . \tag{30}$$

This potential contains all the physical information on the fermion-boson transmutation valid in the particular context of photofission of heavy nuclei, being fixed, at the moment, the region between 40 and 100 MeV.
Note that if $v_k=0$ (i.e. if inside the nucleus there are no pairs contributing to photofission), the transmutational potential from two nucleons to a QD pair becomes equal to minus infinite, in agreement with the results one can deduce from Eq.(19) of Ref. [10]: all the particles are fermions (nucleons). In conclusion, we remark that the state $|NQD>$ of Eq.(27) does not represent, as the state $|BCS>$ of Eq.(25), a redefinition of the vacuum state after having accomplished a Bogoliubov transformation; the state $|NQD>$ represents an excitation state of the nucleus.

V. CONCLUSION

Among the several nucleon pairs that the correlations can arrange into nuclei, the quasi-deuterons are important in specific processes like, for instance, the photofission in the QD energy region. Infact, the empirical-phenomenological QD model describes accurately the behavior of the very different photofission cross sections of the nuclei ^{209}Bi, ^{232}Th and ^{238}U.
This description can be theoretically justified on the basis of the statistical transmutation fermion (nucleon) - boson (QD) due to the transmutational potential studied in this work. The goal has been accomplished by considering, first, a semiclassical kinetic model which describes the distribution functions of nucleons (fermions) and quasi-deuterons (bosons) and the transmutation from one statistical family to the other one. These quantities are given in terms of the nucleon and QD chemical potentials, of the TP or the transmutation rate and are calculated in the framework of the QD model of photofission we introduced in the past and applied to the photofission of ^{209}Bi, ^{232}Th and ^{238}U.
We have found that the TP $\eta_{2N\rightarrow D}(E_\gamma)$ is related to a phase transition in the nuclear system. This has be shown by

means of a Clausius-Clapeyron equation satisfied by the photofission probability.
Finally, we have indicated the existence of a relationship between statistical models, based on the semiclassical kinetic approach and many-body microscopic approaches like the superfluid nuclear model and have shown a microscopic interpretation of the transmutational potential.

Acknowledgments

We would like to thank G. Gervino and G. Lapenta for the stimulating discussions and for the critical reading of the manuscript.

---

[1] P.Ring and P.Schuck, *The nuclear many-body problem*, Springer Verlag, Heidelberg, 1980.
[2] R.Casten, *Nuclear structure from a simple perspective*, Oxford University Press, Oxford,1990.
[3] J.Randrup, Nucl.Phys. A **574**, 133c (1994).
[4] J.Randrup and S.Ayik, Nucl.Phys. A **572**, 489 (1994).
[5] G.Kaniadakis and P.Quarati, Phys. Rev. E **49**, 5103 (1994).
[6] G.Kaniadakis, Phys. Rev. E **49**, 5111 (1994).
[7] G.Kaniadakis, A.Lavagno and P.Quarati, Nucl. Phys. B, in press (1996).
[8] J.L.Levinger, Phys. Lett. B **82**, 181 (1979).
[9] A.Lavagno, *Tesi di Laurea*, University of Turin, 1993, unpublished.
[10] P.P.Delsanto, A. Fubini, F.Murgia and P.Quarati, Zeit. f. Phys. A **342**, 291 (1992).
[11] G.Kaniadakis, U.Lucia and P.Quarati, Int. Jour. Mod. Phys. E **2**, 827 (1993).
[12] J.D.T. Arruda-Neto *et al.*, Phys. Rev. C **51**, R452 (1995).
[13] J.D.T. Arruda-Neto *et al.*, Phys. Rev. C **51**, 751 (1995).
[14] W.Alberico, G.Gervino and A.Lavagno, Phys. Lett. B **321**, 177 (1994).
[15] N.Bianchi *et al*, Phys. Lett. B **321**, 177, (1994).
[16] Th.Frommhold *et al*, Zeit. f. Phys. A **350**, 249 (1994).
[17] D.A.Sims *et al.*, Phys. Rev. C **45**, 479 (1992).
[18] L.Van Hoorebeke *et al.*, Phys. Rev. C **45**, 482 (1992).
[19] J.Ryckebuch *et al.*, Phys. Lett. B **291**, 213 (1992).
[20] V.S. Barashenkov *et al.*, Nucl. Phys. A **231**, 462 (1974).
[21] A.Ilynov *et al.*, Phys. Rev. C **49**, 1420 (1989).
[22] B.J. Cole, N.J. Davidson and H.G. Miller, Phys. Rev C **50**, 1913 (1994).

Table Caption
$F_2(E_\gamma)$ for the three nuclei ^{209}Bi, ^{232}Th, ^{238}U.

Figure Caption
The transmutational potential as a function of the excitation energy for ^{209}Bi, ^{232}Th, ^{238}U ($V_0 = -48\ MeV$).

Table

$E_\gamma\ [MeV]$	40	50	60	70	80	90	100
^{209}Bi	0.0002	0.0003	0.0015	0.004	0.008	0.012	0.019
^{232}Th	0.109	0.142	0.178	0.221	0.269	0.324	0.383
^{238}U	0.183	0.237	0.296	0.396	0.428	0.501	0.578